

NAG C Library Function Document

nag_tsa_arma_filter (g13bac)

1 Purpose

nag_tsa_arma_filter (g13bac) filters a time series by an ARIMA model.

2 Specification

```
void nag_tsa_arma_filter (const double y[], Integer ny, Nag_ArimaOrder *arimaf,
    Nag_ArimaOrder *arimav, const double par[], Integer npar, double cy,
    double b[], Integer nb, NagError *fail)
```

3 Description

From a given series y_1, y_2, \dots, y_n , a new series b_1, b_2, \dots, b_n is calculated using a supplied (filtering) ARIMA model. This model will be one which has previously been fitted to a series x_t with residuals a_t . The equations defining b_t in terms of y_t are very similar to those by which a_t is obtained from x_t . The only dissimilarity is that no constant correction is applied after differencing. This is because the series y_t is generally distinct from the series x_t with which the model is associated, though y_t may be related to x_t . Whilst it is appropriate to apply the ARIMA model to y_t so as to preserve the same relationship between b_t and a_t as exists between y_t and x_t , the constant term in the ARIMA model is inappropriate for y_t . The consequence is that b_t will not necessarily have zero mean.

The equations are precisely:

$$w_t = \nabla^d \nabla_s^D y_t, \quad (1)$$

the appropriate differencing of y_t ; both the seasonal and non-seasonal inverted autoregressive operations are then applied,

$$u_t = w_t - \Phi_1 w_{t-s} - \dots - \Phi_P w_{t-s \times P} \quad (2)$$

$$v_t = u_t - \phi_1 u_{t-1} - \dots - \phi_p u_{t-p} \quad (3)$$

followed by the inverted moving average operations

$$z_t = v_t + \Theta_1 z_{t-s} + \dots + \Theta_Q z_{t-s \times Q} \quad (4)$$

$$b_t = z_t + \theta_1 b_{t-1} + \dots + \theta_q b_{t-q}. \quad (5)$$

Because the filtered series value b_t depends on present and past values y_t, y_{t-1}, \dots , there is a problem arising from ignorance of y_0, y_{-1}, \dots which particularly affects calculation of the early values b_1, b_2, \dots , causing ‘transient errors’. The routine allows two possibilities.

- (i) The equations (1), (2) and (3) are applied from successively later time points so that all terms on their right-hand sides are known, with v_t being defined for $t = (1 + d + s \times D + s \times P), \dots, n$. Equations (4) and (5) are then applied over the same range, taking any values on the right-hand side associated with previous time points to be zero.

This procedure may still however result in unacceptably large transient errors in early values of b_t .

- (ii) The unknown values y_0, y_{-1}, \dots are estimated by backforecasting. This requires that an ARIMA model distinct from that which has been supplied for filtering, should have been previously fitted to y_t .

For efficiency, the user is asked to supply both this ARIMA model for y_t and a limited number of backforecasts which are prefixed to the known values of y_t . Within the routine further backforecasts of y_t , and the series w_t, u_t, v_t in (1), (2), (3) are then easily calculated, and a set of linear equations solved for backforecasts of z_t, b_t for use in (4) and (5) in the case that $q + Q > 0$.

Even if the best model for y_t is not available, a very approximate guess such as

$$y_t = c + e_t$$

or

$$\nabla y_t = e_t$$

can help to reduce the transients substantially.

The backforecasts which need to be prefixed to y_t are of length $Q'_y = q_y + s_y \times Q_y$, where q_y and Q_y are the non-seasonal and seasonal moving average orders and s_y the seasonal period for the ARIMA model of y_t . Thus the user need not carry out the backforecasting exercise if $Q'_y = 0$. Otherwise, the series y_1, y_2, \dots, y_n should be reversed to obtain y_n, y_{n-1}, \dots, y_1 and `nag_tsa_multi_inp_model_forecast` (g13bjc) should be used to forecast Q'_y values, $y_0, \dots, y_{1-Q'_y}$. The ARIMA model used is that fitted to y_t (as a forward series) except that, if $d_y + D_y$ is odd, the constant should be changed in sign (to allow, for example, for the fact that a forward upward trend is a reversed downward trend). The ARIMA model for y_t supplied to the filtering routine must however have the appropriate constant for the forward series.

The series $y_{1-Q'_y}, \dots, y_0, y_1, \dots, y_n$ is then supplied to the routine, and a corresponding set of values returned for b_t .

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

5 Parameters

1: **y[ny]** – const double *Input*

On entry: the Q'_y backforecasts, starting with backforecast at time $1 - Q'_y$ to backforecast at time 0, followed by the time series starting at time 1, where $Q'_y = \mathbf{arimav.q} + \mathbf{arimav.bigq} \times \mathbf{arimav.s}$. If there are no backforecasts, either because the ARIMA model for the time series is not known, or because it is known but has no moving average terms, then the time series starts at the beginning of **y**.

2: **ny** – Integer *Input*

On entry: the total number of backforecasts and time series data points in array **y**.

Constraint: **ny** \geq $\max(1 + Q'_y, \mathbf{npar})$.

3: **arimaf** – Nag_ArimaOrder *Input*

Note: **arimaf** is a NAG defined structure. See Section 2.2.1.1 of the Essential Introduction.

On entry: the orders for the filtering ARIMA model as a pointer to structure of type **Nag_ArimaOrder** with the following members:

p – Integer	<i>Input</i>
d – Integer	<i>Input</i>
q – Integer	<i>Input</i>
bigp – Integer	<i>Input</i>
bigd – Integer	<i>Input</i>
bigq – Integer	<i>Input</i>
s – Integer	<i>Input</i>

These seven members of **arimaf** must specify the orders vector (p, d, q, P, D, Q, s) , respectively, of the ARIMA model for the output noise component.

p , q , P and Q refer, respectively, to the number of autoregressive (ϕ), moving average (θ), seasonal autoregressive (Φ) and seasonal moving average (Θ) parameters.

d , D and s refer, respectively, to the order of non-seasonal differencing, the order of seasonal differencing and the seasonal period.

Constraints:

```

maths> arimaf.p \geq 0;
maths> arimaf.d \geq 0;
maths> arimaf.q \geq 0;
maths> arimaf.bigp \geq 0;
maths> arimaf.bigd \geq 0;
maths> arimaf.bigq \geq 0;
maths> arimaf.s \geq 0;

maths> arimaf.s \neq 1; {\rm arimaf}.p + {\rm arimaf}.q + {\rm arimaf}.bigp + {\rm
arimaf}.bigq > 0. {\rm arimaf}.s = 0 {\rm arimaf}.bigp + {\rm arimaf}.bigd + {\rm
arimaf}.bigq = 0 {\rm arimaf}.s \neq 0 {\rm arimaf}.bigp + {\rm arimaf}.bigd + {\rm
arimaf}.bigq \neq 0

```

4: **arimav** – Nag_ArimaOrder

Input

Note: **arimav** is a NAG defined structure. See Section 2.2.1.1 of the Essential Introduction.

On entry: if available, the orders for the ARIMA model for the series as a pointer to structure of type **Nag_ArimaOrder** with the following members:

p – Integer	<i>Input</i>
d – Integer	<i>Input</i>
q – Integer	<i>Input</i>
bigp – Integer	<i>Input</i>
bigd – Integer	<i>Input</i>
bigq – Integer	<i>Input</i>
s – Integer	<i>Input</i>

These seven members of **arimav** must specify the orders vector (p, d, q, P, D, Q, s) , respectively, of the ARIMA model for the output noise component.

p , q , P and Q refer, respectively, to the number of autoregressive (ϕ), moving average (θ), seasonal autoregressive (Φ) and seasonal moving average (Θ) parameters.

d , D and s refer, respectively, to the order of non-seasonal differencing, the order of seasonal differencing and the seasonal period.

If no ARIMA model for the series is to be supplied **arimav** should be set to a **NULL** pointer.

Constraints:

```

maths> arimav.p \geq 0;
maths> arimav.d \geq 0;
maths> arimav.q \geq 0;
maths> arimav.bigp \geq 0;
maths> arimav.bigd \geq 0;
maths> arimav.bigq \geq 0;

maths> arimav.s \geq 0; {\rm arimav}.s \neq 1. {\rm arimav}.s = 0 {\rm arimav}.bigp + {\rm
arimav}.bigd + {\rm arimav}.bigq = 0 {\rm arimav}.s \neq 0 {\rm arimav}.bigp + {\rm
arimav}.bigd + {\rm arimav}.bigq \neq 0

```

- 5: **par**[**npar**] – const double *Input*
On entry: the parameters of the filtering model, followed by the parameters of the ARIMA model for the time series, if supplied. Within each model the parameters are in the standard order of non-seasonal AR and MA followed by seasonal AR and MA.
- 6: **npar** – Integer *Input*
On entry: the total number of parameters held in array **par**.
Constraints: **arimav** is **NULL****npar** = **arimaf.p** + **arimaf.q** + **arimaf.bigp** + **arimaf.bigq**
arimav is not **NULL****npar** = **arimaf.p** + **arimaf.q** + **arimaf.bigp** + **arimaf.bigq** + **arimav.p** + **arimav.q** + **arimav.bigp** + **arimav.bigq**
Note: the first constraint (i.e., **arimaf.p** + **arimaf.q** + **arimaf.bigp** + **arimaf.bigq** > 0) on the orders of the filtering model, in parameter **arimaf**, ensures that **npar** > 0.
- 7: **cy** – double *Input*
On entry: if the ARIMA model is known, **cy** must specify the constant term of the ARIMA model for the time series. If this model is not known, then **cy** is not used.
- 8: **b**[**nb**] – double *Output*
On exit: the filtered output series. If the ARIMA model for the time series was known, and hence Q'_y backforecasts were supplied in **y**, then **b** contains Q'_y ‘filtered’ backforecasts followed by the filtered series. Otherwise, the filtered series begins at the start of **b** just as the original series began at the start of **y**. In either case, if the value of the series at time t is held in **y**[$t - 1$], then the filtered value at time t is held in **b**[$t - 1$].
- 9: **nb** – Integer *Input*
On entry: the dimension of the array **b** as declared in the function from which `nag_tsa_arma_filter` (g13bac) is called. In addition to holding the returned filtered series, **b** is also used as an intermediate work array if the ARIMA model for the time series was known.
Constraints: **arimav** is not **NULL****nb** \geq **ny** + $\max(k_3, k_1 + k_2)$ **arimav** is **NULL****nb** \geq **ny**
where

$$k_1 = \mathbf{arimaf.p} + \mathbf{arimaf.bigp} \times \mathbf{arimaf.s};$$

$$k_2 = \mathbf{arimaf.d} + \mathbf{arimaf.bigd} \times \mathbf{arimaf.s};$$

$$k_3 = \mathbf{arimaf.q} + \mathbf{arimaf.bigq} \times \mathbf{arimaf.s}.$$
- 10: **fail** – NagError * *Input/Output*
The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **ny** is too small to carry out requested filtering: **ny** = $\langle value \rangle$.

NE_CONSTRAINT

General constraint: **arimaf.p** \geq 0 and

General constraint: **arimaf.d** \geq 0 and

General constraint: **arimaf.q** \geq 0 and

General constraint: **arimaf.bigp** \geq 0 and

General constraint: **arimaf.bigd** \geq 0 and

General constraint: **arimaf.bigq** \geq 0 and

General constraint: $\mathbf{arimaf.s} \geq 0$ and

General constraint: $\mathbf{arimaf.s} \neq 1$ and

General constraint: $\mathbf{arimaf.p} + \mathbf{arimaf.q} + \mathbf{arimaf.bigp} + \mathbf{arimaf.bigq} > 0$ and

General constraint: $\mathbf{arimav.p} \geq 0$ and

General constraint: $\mathbf{arimav.d} \geq 0$ and

General constraint: $\mathbf{arimav.q} \geq 0$ and

General constraint: $\mathbf{arimav.bigp} \geq 0$ and

General constraint: $\mathbf{arimav.bigd} \geq 0$ and

General constraint: $\mathbf{arimav.bigq} \geq 0$ and

General constraint: $\mathbf{arimav.s} \geq 0$ and

General constraint: $\mathbf{arimav.s} \neq 1$ and

General constraint: if $\mathbf{arimaf.s} = 0$, $\mathbf{arimaf.bigp} + \mathbf{arimaf.bigd} + \mathbf{arimaf.bigq} = 0$.

General constraint: if $\mathbf{arimaf.s} \neq 0$, $\mathbf{arimaf.bigp} + \mathbf{arimaf.bigd} + \mathbf{arimaf.bigq} \neq 0$.

General constraint: if $\mathbf{arimav.s} = 0$, $\mathbf{arimav.bigp} + \mathbf{arimav.bigd} + \mathbf{arimav.bigq} = 0$.

General constraint: if $\mathbf{arimav.s} \neq 0$, $\mathbf{arimav.bigp} + \mathbf{arimav.bigd} + \mathbf{arimav.bigq} \neq 0$.

NE_ARRAY_SIZE

The array \mathbf{b} is too small. Minimum required size: $\langle value \rangle$.

NE_CONSTRAINT

On entry, \mathbf{npar} is inconsistent with \mathbf{arimaf} and \mathbf{arimav} : $\mathbf{npar} = \langle value \rangle$.

NE_INIT_FILTER

The initial values of the filtered series are indeterminate for the given models.

NE_ORDERS_ARIMA

The orders vector for the ARIMA model is invalid.

On entry, \mathbf{arimaf} or \mathbf{arimav} is invalid.

NE_ORDERS_FILTER

The orders vector for the filtering model is invalid.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

Accuracy and stability are high except when the MA parameters are close to the invertibility boundary.

8 Further Comments

The time taken by the routine is approximately proportional to

$$ny \times (\text{arimaf.p} + \text{arimaf.q} + \text{arimaf.bigp} + \text{arimaf.bigq}),$$

with an appreciable fixed increase if an ARIMA model is supplied for the time series.

9 Example

The example program reads a time series of length 296. It reads the univariate ARIMA (4,0,2,0,0,0) model and the ARIMA filtering (3,0,0,0,0,0) model for the series. Two initial backforecasts are required and these are calculated by a call to `nag_tsa_multi_inp_model_forecast` (g13bjc). The backforecasts are inserted at the start of the series and `nag_tsa_arma_filter` (g13bac) is called to perform the calculations.

9.1 Program Text

```

/* nag_tsa_arma_filter (g13bac) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    double a1, a2, cx, cy;
    Integer i, idd, ii, ij, iqxd,
            j, k, n, nb, ni, nmr, npar, nparx, nx, ny,
            nser, npara, tdxy, tdmrx, ldparx, tdpax;
    Integer exit_status=0;

    /* Arrays */
    double *b = 0, *fsd = 0, *fva = 0, *par = 0, *parx = 0,
           *x = 0, *y = 0, *rms=0, *parxx=0;
    Integer mr[14], mrx[7], *mrxx=0;
    Nag_ArimaOrder arimaj, arimaf, arimav;
    Nag_TransfOrder transfj;
    Nag_G13_Opt options;
    NagError fail;

    INIT_FAIL(fail);
    exit_status = 0;

    /* Initialise the options structure used by g13bjc */
    g13bxc(&options);

    Vprintf("g13bac Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[^\\n] ");
    Vscanf("%ld%*[^\\n] ", &nx);

    if (nx > 0)
    {
        /* Allocate array x */
        if (!(x = NAG_ALLOC(nx+2, double)))
        {
            Vprintf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
    }
}

```

```

for (i = 1; i <= nx; ++i)
  Vscanf("%lf", &x[i-1]);
Vscanf("%*[\n] ");

/* Read univariate Arima for series */
for (i = 1; i <= 7; ++i)
  Vscanf("%ld", &mrx[i-1]);
Vscanf("%*[\n] ");

Vscanf("%lf%*[\n] ", &cx);

nparx = mrx[0] + mrx[2] + mrx[3] + mrx[5];

arimaj.p = mrx[0];
arimaj.d = mrx[1];
arimaj.q = mrx[2];
arimaj.bigp = mrx[3];
arimaj.bigd = mrx[4];
arimaj.bigq = mrx[5];
arimaj.s = mrx[6];

nser = 1;

if (nparx > 0)
  {
  /* Allocate array parx */
  if (!(parx = NAG_ALLOC(nparx+nser, double)))
    {
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
    }

  for (i = 1; i <= nparx; ++i)
    Vscanf("%lf", &parx[i-1]);
  Vscanf("%*[\n] ");

  /* Read model by which to filter series */
  for (i = 1; i <= 7; ++i)
    Vscanf("%ld", &mr[i-1]);
  Vscanf("%*[\n] ");

  arimaf.p = mr[0];
  arimaf.d = mr[1];
  arimaf.q = mr[2];
  arimaf.bigp = mr[3];
  arimaf.bigd = mr[4];
  arimaf.bigq = mr[5];
  arimaf.s = mr[6];

  npar = mr[0] + mr[2] + mr[3] + mr[5];
  if (npar > 0)
    {
    /* Allocate array par */
    if (!(par = NAG_ALLOC(npar + nparx, double)))
      {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
      }
    for (i = 1; i <= npar; ++i)
      Vscanf("%lf", &par[i-1]);
    Vscanf("%*[\n] ");

    /* Initially backforecast QY values */
    /* (1) Reverse series in situ */
    n = nx / 2;
    ni = nx;
    for (i = 1; i <= n; ++i)
      {

```

```

        a1 = x[i-1];
        a2 = x[ni-1];
        x[i-1] = a2;
        x[ni-1] = a1;
        --ni;
    }
    idd = mrx[1] + mrx[4];
    /* (2) Possible sign reversal for ARIMA constant */
    if (idd % 2 != 0)
        cx = -cx;

    /* (3) Calculate number of backforecasts required */
    iqxd = mrx[2] + mrx[5] * mrx[6];

    /* Calculate series length */
    ny = nx + iqxd;

    /* Allocate array y */
    if (!(y = NAG_ALLOC(ny, double)))
    {
        Vprintf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    if (iqxd != 0)
    {
        /* Allocate arrays fsd, fva and st. */
        if ( !(fsd = NAG_ALLOC(iqxd, double)) ||
            !(fva = NAG_ALLOC(iqxd, double)) )
        {
            Vprintf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }
        /* (4) Set up parameter list for call to forecast
         * routine g13bjc
         */
        npara = nparx+nser;
        parx[npara-1] = cx;
        tdxy = nser;
        tdmrx = nser-1;
        ldparx = nser-1;
        tdpax = nser-1;
        if ( !(rms = NAG_ALLOC(nser, double)) ||
            !(parxx = NAG_ALLOC(nser, double)) ||
            !(mrxx = NAG_ALLOC(7*nser, Integer)) )
        {
            Vprintf("Allocation failure\n");
            exit_status = -1;
            goto END;
        }

        g13byc(nser, &transfj, &fail);
        if (fail.code != NE_NOERROR)
        {
            Vprintf("Error from g13byc.\n%s\n", fail.message);
            exit_status = 1;
            goto END;
        }

        rms[0] = 0;
        transfj.nag_b = 0;
        transfj.nag_q = 0;
        transfj.nag_p = 0;
        transfj.nag_r = 1;
        for (i=1; i<=7; ++i)
            mrxx[i-1] = 0;
        parxx[0] = 0;

        /* Tell g13bjc not to print parameters on entry */

```



```

options.list = FALSE;

g13bjc(&arimaj, nser, &transfj, parx, npara, nx, iqxd, x,
      tdxy, rms, mrx, tdmrx, parxx, ldparx, tdpax,
      fva, fsd, &options, &fail);
if (fail.code != NE_NOERROR)
  {
  Vprintf("Error from g13bjc.\n%s\n", fail.message);
  exit_status = 1;
  goto END;
  }

j = iqxd;
for (i = 1; i <= iqxd; ++i)
  {
  y[i-1] = fva[j-1];
  --j;
  }

/* Move series into y */
j = iqxd + 1;
k = nx;
for (i = 1; i <= nx; ++i)
  {
  if (j > 305)
    goto END;
  y[j-1] = x[k-1];
  ++j;
  --k;
  }

/* Move ARIMA for series into mr */
for (i = 1; i <= 7; ++i)
  mr[i+6] = mrx[i-1];

arimav.p = mr[7];
arimav.d = mr[8];
arimav.q = mr[9];
arimav.bigp = mr[10];
arimav.bigd = mr[11];
arimav.bigq = mr[12];
arimav.s = mr[13];

/* Move parameters of ARIMA for y into par */
for (i = 1; i <= nparx; ++i)
  par[npar+i-1] = parx[i-1];
npar += nparx;

/* Move constant and reset sign reversal */
cy = cx;
if (idd % 2 != 0)
  cy = -cy;

/* Set parameters for call to filter routine g13bac */
nmr = 14;
nb = ny + MAX(mr[2] + mr[5] * mr[6],
              mr[0] + mr[1] + (mr[3] + mr[4]) * mr[6]);

/* Allocate array b */
if ( !(b = NAG_ALLOC(nb, double)) )
  {
  Vprintf("Allocation failure\n");
  exit_status = -1;
  goto END;
  }

/* Filter series by call to g13bac */
g13bac(y, ny, &arimaf, &arimav, par, npar, cy, b,
      nb, &fail);
if (fail.code != NE_NOERROR)

```

```

    {
        Vprintf("Error from g13bac.\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    Vprintf("\n");
    Vprintf("          Original          Filtered\n");
    Vprintf("Backforecasts  y-series      series\n");
    if (iqxd != 0)
    {
        ij = -iqxd;
        for (i = 1; i <= iqxd; ++i)
        {
            Vprintf("%8ld%17.4f%15.4f\n", ij, y[i-1], b[i-1]);
            ++ij;
        }
    }

    Vprintf("\n");
    Vprintf("          Filtered          Filtered          "
           "Filtered          "
           "Filtered          Filtered\n");
    Vprintf("          series          series          "
           "          series          series\n");
    for (i = iqxd + 1; i <= ny; i += 4)
    {
        for (ii = i; ii <= MIN(ny, i+3); ++ii)
        {
            Vprintf("%5ld", ii-iqxd);
            Vprintf("%9.4f  ", b[ii-1]);
        }
        Vprintf("\n");
    }
}
}
}

END:

/* Free the options structure used by g13bjc */
g13xzc(&options);

if (b) NAG_FREE(b);
if (fsd) NAG_FREE(fsd);
if (fva) NAG_FREE(fva);
if (par) NAG_FREE(par);
if (parx) NAG_FREE(parx);
if (x) NAG_FREE(x);
if (y) NAG_FREE(y);
if (rms) NAG_FREE(rms);
if (parxx) NAG_FREE(parxx);
if (mrxx) NAG_FREE(mrxx);

return exit_status;
}

```

9.2 Program Data

g13bac Example Program Data

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```

53.8 53.6 53.5 53.5 53.4 53.1 52.7 52.4 52.2 52.0 52.0 52.4 53.0 54.0 54.9 56.0
56.8 56.8 56.4 55.7 55.0 54.3 53.2 52.3 51.6 51.2 50.8 50.5 50.0 49.2 48.4 47.9
47.6 47.5 47.5 47.6 48.1 49.0 50.0 51.1 51.8 51.9 51.7 51.2 50.0 48.3 47.0 45.8
45.6 46.0 46.9 47.8 48.2 48.3 47.9 47.2 47.2 48.1 49.4 50.6 51.5 51.6 51.2 50.5
50.1 49.8 49.6 49.4 49.3 49.2 49.3 49.7 50.3 51.3 52.8 54.4 56.0 56.9 57.5 57.3
56.6 56.0 55.4 55.4 56.4 57.2 58.0 58.4 58.4 58.1 57.7 57.0 56.0 54.7 53.2 52.1
51.6 51.0 50.5 50.4 51.0 51.8 52.4 53.0 53.4 53.6 53.7 53.8 53.8 53.8 53.3 53.0
52.9 53.4 54.6 56.4 58.0 59.4 60.2 60.0 59.4 58.4 57.6 56.9 56.4 56.0 55.7 55.3
55.0 54.4 53.7 52.8 51.6 50.6 49.4 48.8 48.5 48.7 49.2 49.8 50.4 50.7 50.9 50.7
50.5 50.4 50.2 50.4 51.2 52.3 53.2 53.9 54.1 54.0 53.6 53.2 53.0 52.8 52.3 51.9

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51.6 51.6 51.4 51.2 50.7 50.0 49.4 49.3 49.7 50.6 51.8 53.0 54.0 55.3 55.9 55.9
54.6 53.5 52.4 52.1 52.3 53.0 53.8 54.6 55.4 55.9 55.9 55.2 54.4 53.7 53.6 53.6
53.2 52.5 52.0 51.4 51.0 50.9 52.4 53.5 55.6 58.0 59.5 60.0 60.4 60.5 60.2 59.7
59.0 57.6 56.4 55.2 54.5 54.1 54.1 54.4 55.5 56.2 57.0 57.3 57.4 57.0 56.4 55.9
55.5 55.3 55.2 55.4 56.0 56.5 57.1 57.3 56.8 55.6 55.0 54.1 54.3 55.3 56.4 57.2
57.8 58.3 58.6 58.8 58.8 58.6 58.0 57.4 57.0 56.4 56.3 56.4 56.4 56.0 55.2 54.0
53.0 52.0 51.6 51.6 51.1 50.4 50.0 50.0 52.0 54.0 55.1 54.5 52.8 51.4 50.8 51.2
52.0 52.8 53.8 54.5 54.9 54.9 54.8 54.4 53.7 53.3 52.8 52.6 52.6 53.0 54.3 56.0
57.0 58.0 58.6 58.5 58.3 57.8 57.3 57.0
  4      0      2      0      0      0      0
0.000
2.420      -2.380      1.160      -0.230      0.310      -0.470
  3      0      0      0      0      0
1.970      -1.370      0.340
    
```

9.3 Program Results

g13bac Example Program Results

Backforecasts	Original y-series	Filtered series
-2	49.9807	3.4222
-1	52.6714	3.0809

Filtered series	Filtered series	Filtered series	Filtered series
1 2.9813	2 2.7803	3 3.7057	4 3.2450
5 3.0760	6 3.0070	7 3.0610	8 3.1720
9 3.1170	10 3.0360	11 3.2580	12 3.4520
13 3.3320	14 3.6980	15 3.3140	16 3.8070
17 3.3330	18 2.9580	19 3.2800	20 3.0960
21 3.2270	22 3.0830	23 2.6410	24 3.1870
25 2.9910	26 3.1110	27 2.8460	28 3.0240
29 2.7030	30 2.6130	31 2.8060	32 2.9560
33 2.8170	34 2.8950	35 2.8510	36 2.9160
37 3.2530	38 3.3050	39 3.1830	40 3.3760
41 2.9730	42 2.8610	43 3.0490	44 2.8420
45 2.3190	46 2.3660	47 2.9410	48 2.3810
49 3.3420	50 2.9340	51 3.1800	52 2.9230
53 2.6470	54 2.8860	55 2.5310	56 2.6200
57 3.4170	58 3.4940	59 3.2590	60 3.1310
61 3.1420	62 2.6710	63 2.8990	64 2.8180
65 3.2150	66 2.8800	67 2.9610	68 2.8800
69 3.0020	70 2.8930	71 3.1210	72 3.2210
73 3.2040	74 3.5360	75 3.7520	76 3.5630
77 3.7260	78 3.1560	79 3.6310	80 2.9380
81 3.1480	82 3.4490	83 3.1400	84 3.7380
85 4.1200	86 3.1540	87 3.7480	88 3.3280
89 3.3640	90 3.3400	91 3.3950	92 3.0720
93 3.0050	94 2.8520	95 2.7810	96 3.1950
97 3.2490	98 2.6370	99 3.0080	100 3.2410
101 3.5570	102 3.2080	103 3.0880	104 3.3980
105 3.1660	106 3.1960	107 3.2460	108 3.2870
109 3.1590	110 3.2620	111 2.7280	112 3.4130
113 3.2190	114 3.6750	115 3.8550	116 4.0100
117 3.5380	118 3.8440	119 3.4660	120 3.0640
121 3.4780	122 3.1140	123 3.5300	124 3.2400
125 3.3630	126 3.2610	127 3.3020	128 3.1150
129 3.3280	130 2.8730	131 3.0800	132 2.8390
133 2.6570	134 3.0260	135 2.4580	136 3.2600
137 2.8380	138 3.2150	139 3.1140	140 3.1050
141 3.1400	142 2.9100	143 3.1370	144 2.7500
145 3.1160	146 3.0680	147 2.8590	148 3.3840
149 3.5500	150 3.4160	151 3.1770	152 3.3390
153 3.0190	154 3.1780	155 3.0110	156 3.1940
157 3.2680	158 3.0500	159 2.8060	160 3.1850
161 3.0560	162 3.2690	163 2.7940	164 3.0900
165 2.7100	166 2.7890	167 2.9510	168 3.2440
169 3.2570	170 3.4360	171 3.4450	172 3.3780
173 3.3520	174 3.9180	175 2.9190	176 3.1780

177	2.2580	178	3.5150	179	2.8010	180	3.6030
181	3.2610	182	3.5300	183	3.3270	184	3.4420
185	3.5240	186	3.2720	187	3.1110	188	2.8240
189	3.2330	190	3.1500	191	3.5710	192	3.0810
193	2.7820	194	2.9040	195	3.2350	196	2.7970
197	3.1320	198	3.1680	199	4.5210	200	2.6650
201	4.6870	202	3.9470	203	3.2220	204	3.3410
205	3.9950	206	3.4820	207	3.3630	208	3.4550
209	3.2950	210	2.6910	211	3.4600	212	2.9440
213	3.4400	214	3.1830	215	3.4200	216	3.4100
217	4.0550	218	2.9990	219	3.8250	220	3.1340
221	3.5010	222	3.0430	223	3.2660	224	3.3660
225	3.2650	226	3.3720	227	3.2880	228	3.5470
229	3.6840	230	3.3100	231	3.6790	232	3.1780
233	2.9360	234	2.7910	235	3.8020	236	2.6100
237	4.1690	238	3.7460	239	3.4560	240	3.3910
241	3.5820	242	3.6220	243	3.4870	244	3.5770
245	3.4240	246	3.3960	247	3.1220	248	3.4300
249	3.4580	250	3.0280	251	3.7660	252	3.3770
253	3.2470	254	3.0180	255	2.9720	256	2.8000
257	3.2040	258	2.8020	259	3.4100	260	3.1680
261	2.4600	262	2.8810	263	3.1750	264	3.1740
265	4.8640	266	3.0600	267	2.9600	268	2.2530
269	2.5620	270	3.3150	271	3.3480	272	3.5900
273	3.2560	274	3.2320	275	3.6160	276	3.1700
277	3.2890	278	3.1200	279	3.3300	280	2.9910
281	2.9420	282	3.4070	283	2.8720	284	3.3470
285	3.1920	286	3.4880	287	4.0680	288	3.7550
289	3.0510	290	3.9680	291	3.3900	292	3.1380
293	3.6170	294	3.1700	295	3.4150	296	3.4830
