

NAG C Library Function Document

nag_prob_non_central_students_t (g01gbc)

1 Purpose

nag_prob_non_central_students_t (g01gbc) returns the lower tail probability for the non-central Student's t -distribution.

2 Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_prob_non_central_students_t (double t, double df, double delta,
                                         double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the non-central Student's t -distribution with ν degrees of freedom and non-centrality parameter δ , $P(T \leq t : \nu; \delta)$ is defined by:

$$P(T \leq t : \nu; \delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2} dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma(\frac{1}{2}\nu) 2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}$$

The probability is computed in one of two ways,

(a) when $t = 0.0$, the relationship to the normal is used

$$P(T \leq t : \nu; \delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2} du;$$

(b) otherwise the series expansion described in Amos (1964) (equation 9) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 Parameters

- | | | |
|----|--|--------------|
| 1: | t – double | <i>Input</i> |
| | <i>On entry:</i> the deviate from the Student's t -distribution with ν degrees of freedom, t . | |
| 2: | df – double | <i>Input</i> |
| | <i>On entry:</i> the degrees of freedom of the Student's t -distribution, ν . | |
| | <i>Constraint:</i> df \geq 1.0. | |
| 3: | delta – double | <i>Input</i> |
| | <i>On entry:</i> the non-centrality parameter of the Students t -distribution, δ . | |

- 4: **tol** – double *Input*
On entry: the absolute accuracy required by the user in the results.
 If `nag_prob_non_central_students_t` is entered with **tol** greater than or equal to 1.0 or less than $10 \times \textit{machine precision}$, then the value of $10 \times \textit{machine precision}$ is used instead.
- 5: **max_iter** – Integer *Input*
On entry: the maximum number of terms that are used in each of the summations.
Suggested value: 100. See Section 6 for further comments.
Constraint: **max_iter** ≥ 1 .
- 6: **fail** – NagError * *Input/Output*
 The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_REAL_ARG_LT

On entry, **df** must not be less than 1.0: **df** = *<value>*.

NE_INT_ARG_LT

On entry, **max_iter** must not be less than 1: **max_iter** = *<value>*.

NE_SERIES

One of the series has failed to converge with **df** = *<value>* and **max_iter** = *<value>*. Reconsider the requested tolerance and/or the maximum number of iterations.

NE_PROBABILITY

The probability is too small to calculate accurately.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

The rate of convergence of the series depends, in part, on the quantity: $t^2/(t^2 + \nu)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the t -distribution to the F -distribution can be used:

$$F = T^2, \quad \lambda = \delta^2, \quad \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to `nag_prob_non_central_f_dist` (g01gdc).

Note: this routine only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

6.1 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than **tol**. There may also be some slight loss of accuracy due to calculation of gamma functions. For large values of $\delta > 50$ there may be significant loss of accuracy.

6.2 References

Amos D E (1964) Representations of the central and non-central t -distributions *Biometrika* **51** 451–458

7 See Also

nag_prob_non_central_students_t (g01gbc)

8 Example

Values from, and degrees of freedom for and non-centrality parameter of the non-central Student's t -distributions are read, the lower tail probabilities calculated and all these values printed until the end of data is reached.

8.1 Program Text

```

/* nag_prob_non_central_students_t (g01gbc) Example Program.
 *
 * Copyright 1999 Numerical Algorithms Group.
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{
    double delta, df, prob, t, tol;
    Integer max_iter;
    Integer exit_status = 0;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("g01gbc Example Program Results\n\n");

    /* Skip heading in data file */
    Vscanf("%*[\n]");

    Vprintf("      t      df      delta      prob\n\n");
    tol = 5e-6;
    max_iter = 50;
    while ((scanf("%lf %lf %lf %*[\n]", &t, &df, &delta)) != EOF)
    {
        prob = g01gbc(t, df, delta, tol, max_iter, &fail);
        if (fail.code == NE_NOERROR)
            Vprintf(" %8.3f%8.3f%8.3f%8.4f\n", t, df, delta, prob);
        else
        {
            Vprintf("Error from g01gbc.\n%s\n", fail.message);
            exit_status=1;
            goto END;
        }
    }
    END:
    return exit_status;
}

```

8.2 Program Data

g01gbc Example Program Data

-1.528	20.0	2.0	:t	df	delta
-0.188	7.5	1.0	:t	df	delta
1.138	45.0	0.0	:t	df	delta

8.3 Program Results

g01gbc Example Program Results

t	df	delta	prob
-1.528	20.000	2.000	0.0003
-0.188	7.500	1.000	0.1189
1.138	45.000	0.000	0.8694
