

NAG C Library Function Document

nag_prob_studentized_range (g01emc)

1 Purpose

nag_prob_studentized_range (g01emc) returns the probability associated with the lower tail of the distribution of the Studentized range statistic.

2 Specification

double nag_prob_studentized_range (double **q**, double **v**, Integer **ir**, NagError ***fail**)

3 Description

The externally Studentized range, q , for a sample, x_1, x_2, \dots, x_r , is defined as:

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i 's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \dots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{largest}$ and $\bar{T}_{smallest}$, divided by the square root of the mean-square experimental error, MS_{error} , over the number of observations in each group, n , i.e.,

$$q = \frac{\bar{T}_{largest} - \bar{T}_{smallest}}{\sqrt{MS_{error}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman-Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - qx)]^{r-1} dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t) dt.$$

The above two-dimensional integral is evaluated using numerical quadrature with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - q)]^{r-1} dy.$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32** (2) 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

- | | | |
|----|---|---------------------|
| 1: | q – double | <i>Input</i> |
| | <i>On entry:</i> the Studentized range statistic, q . | |
| | <i>Constraint:</i> $q > 0.0$. | |
| 2: | v – double | <i>Input</i> |
| | <i>On entry:</i> the number of degrees of freedom for the experimental error, v . | |
| | <i>Constraint:</i> $v \geq 1.0$. | |
| 3: | ir – Integer | <i>Input</i> |
| | <i>On entry:</i> the number of groups, r . | |
| | <i>Constraint:</i> $ir \geq 2$. | |
| 4: | fail – NagError * | <i>Input/Output</i> |
| | The NAG error parameter (see the Essential Introduction). | |

6 Error Indicators and Warnings

NE_INT

On entry, **ir** = $\langle value \rangle$.
 Constraint: $ir \geq 2$.

NE_ACCURACY

Warning - There is some doubt as to whether full accuracy has been achieved.

NE_REAL

On entry, **q** = $\langle value \rangle$.
 Constraint: $q > 0.0$.
 On entry, **v** = $\langle value \rangle$.
 Constraint: $v \geq 1.0$.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless **fail.code** = NE_ACCURACY. When **fail.code** = NE_ACCURACY it is usual that the returned value will be a good estimate of the true value.

8 Further Comments

None.

9 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of q , ν and r .

9.1 Program Text

```

/* nag_prob_studentized_range (g01emc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    /* Scalars */
    double q, v, valp;
    Integer exit_status, i, ifail, ir;
    NagError fail;

    Vprintf("g01emc Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[\n] ");

    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("\n%s\n\n", " q          v      ir      Quantile ");
    for (i = 1; i <= 3; ++i)
    {
        Vscanf("%lf%lf%ld%*[\n] ", &q, &v, &ir);
        ifail = -1;
        valp = g01emc(q, v, ir, &fail);

        if (fail.code == NE_NOERROR)
        {
            Vprintf("%7.4f%2s%4.1f%1s%3ld%1s%10.4f\n", q, "",
                    v, "", ir, "", valp);
        }
        else
        {
            Vprintf("Error from g01emc.\n%s\n", fail.message);
            exit_status = 1;
        }
    }
    return exit_status;
}

```

9.2 Program Data

```

g01emc Example Program Data
4.6543 10.0 5
2.8099 60.0 12
4.2636 5.0 4

```

9.3 Program Results

g01emc Example Program Results

q	v	ir	Quantile
4.6543	10.0	5	0.9500
2.8099	60.0	12	0.3000
4.2636	5.0	4	0.9000
